# A novel determination of the critical temperature

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A key to an analyses of nuclear multifragmentation data leading to the nuclear matter phase diagram [1] was Fisher's droplet model [2]. At coexistence Fisher's model gives the temperature $T$ cluster yields as

$$n_s(T) \square\, g(s)\exp(-ws/T) \tag{1}$$

where $s$ is the cluster's surface area, $g(s)$ is proportional to the cluster's degeneracy, $w$ is the surface tension.

Based on the combinatorics of two dimensional clusters Fisher suggested $g(s)$ would be given by

$$g(s) \square\, s^{-x}\exp(\square s) \tag{2}$$

where $x$ is set by the Euclidian dimension and $\square$ is the surface entropy tension. Figure. 1 shows Eq. (2) describes a direct counting of these cluster combinatorics [3].

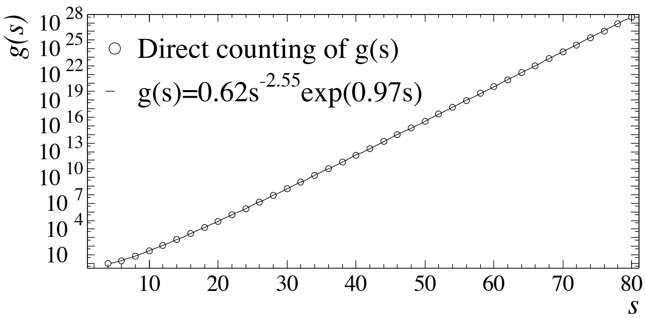


FIG. 1: Degeneracy factor for polygons on the square lattice.

Inserting Eq. (2) into Eq. (1) yields

$$n_s(T) \square\, s^{-x}\exp\left[-s\left(w - T\square\right)/T\right] \tag{3}$$

The exponential's argument is the free energy $\square G$. At the critical temperature $T_c$ $\square G = 0$ and $T_c = w/\square$. For the two dimensional Ising model (isomorphous with the lattice gas) $w = 2$ and with the Fig. 1 parameters $Tc = 2.06$, within 10% of Onsager's value $T_c = 2.26915\ldots$ [5].

To make a better estimate of $T_c$ we think of an initial configuration of a liquid drop with $A_0$ constituents and surface $s_0$ and a final state of a cluster of $A$ constituents and surface $s$ and its complement: a liquid drop of $A_0 - A$ constituents and surface $s_c$. This assumes stochastic cluster formation and is supported by the Ising cluster's Poissonian nature [4]. Now

$$\begin{aligned} \square G &= \square E - T\square S + p\square V \\ &= e_0\left[A + (A_0 - A) - A_0\right] + w(s + s_c - s_0) \\ &- T\left[(\ln g(s) + \ln g(s_c) - \ln g(s_0)\right] + p\square V \end{aligned} \tag{4}$$

where $e_0$ is the volume energy coefficient, $p$ is the pressure and $\square V$ is the volume change. All terms $\square$ $A$ cancel. In the large liquid drop limit $s_c \approx s_0$ and $\ln g(s_c) \approx \ln g(s_0)$ leaving only the cluster's contribution to the $\square G$. The volume change for the lattice gas is

$$\square V = \left[A + (A_0 - A) - A_0\right] + l(s + s_c - s_0) \tag{5}$$

where $l$ is the interaction range between two constituents, one spacing on a lattice: $l = 1$. In in the large drop limit the first part of Eq. (5) cancels and the second part depends only on the cluster's surface so Eq. (1) becomes

$$\begin{aligned} n_s(T) &\square\, g(s)\exp(-ws/T)\exp(2pls/T) \\ &\square\, s^{-x}\exp\left[-s\left(w + 2pl - T\square\right)/T\right]. \end{aligned} \tag{6}$$

The factor of two arises from moving the cluster from the liquid to the vapor. The free energy vanishes at the critical point so $T_c = (w + 2p_c l)/\square$ with $p_c \approx 0.11$ [6] $T_c = 2.29$, within 1% of the Onsager value.

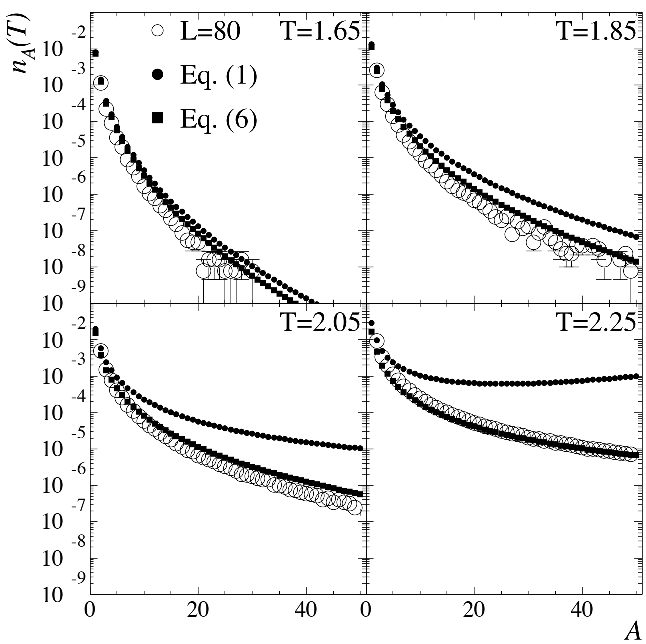


FIG. 2: Ising cluster yields compared to Eq. (1) and (6).

Equation (6) also provides a better description of Ising cluster yields than Eq. (1). Figure 2 shows the Ising yields ($n_A(T) = \square_s n_{A,s}(T)$) of a two dimensional square lattice of side $L = 80$ and the predictions of Eq. (6) and (1) with *no fit parameters*.


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